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Multi-Period Resource Allocation for Estimating Project Costs in Competitive Bidding

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Abstract

To establish a profit-making strategy in competitive bidding for project contracts, it is crucial for contractors to estimate project costs accurately. Although allocating a large amount of resources to cost estimates allows contractors to prepare more accurate estimates, there is usually a limit to available resources in practice. To maximize a contractor's expected profit, this paper develops a multi-period resource allocation method for estimating project costs in a sequential competitive bidding situation. Our resource allocation model is posed as a mixed integer linear programming problem by making a piecewise linear approximation of the expected profit functions. Numerical experiments examine the characteristics of the optimal resource allocation and demonstrate the effectiveness of our resource allocation method.

Keywords: Resource allocation, Cost estimation, Project management, Competitive bidding, Mixed-integer linear programming

1 Introduction

Competitive bidding is widely used to choose contractors under the most favorable conditions. As a typical case, contractors, who have received an invitation from a potential client, submit bid prices. If a contractor's bid price is the lowest among the competitors', s/he will win the project contract. The bid price is then paid to the contractor, and s/he will execute the project as the client has requested. If the actual project cost is kept below the bid price, this project contract will be profitable for the contractor. Otherwise, s/he will suffer a loss on account of cost overruns. In such a process, a contractor's profit is highly dependent on his/her bidding strategy. Consequently, since the seminal work by Friedman (1956), a considerable number of studies have dealt with competitive bidding strategies (see, e.g., Engelbrecht-Wiggans, 1980; King and Mercer, 1988; Rothkopf and Harstad, 1994; Stark and Rothkopf, 1979).

In competitive bidding, contractors estimate the cost of completing a project and then determine the bid price. Accordingly, the bid price is markedly affected by the inaccuracies in the estimated cost, and it is crucial for contractors to estimate the project cost accurately. Naert and Weverbergh (1978) are the first to consider the uncertainty about the estimated cost in a competitive bidding model. Their model shows that the expected profit decreases as the uncertainty about the estimated cost increases. King and Mercer (1990) derive analytical solutions to the competitive bidding models for various distributions of the estimated cost. Takano et al. (2014) establish "sequential" competitive bidding strategies considering inaccurate cost estimates, whereas other studies (King and Mercer, 1990; Naert and Weverbergh, 1978) discuss only "one-shot" bidding.

As pointed out by Towler and Sinnott (2012), the accuracy of the cost estimation is positively correlated with the man-hours (MH) thrown into making the estimate. In addition, Christensen and Dysert (1997) devise a cost estimate classification matrix that shows a clear relationship between the accuracy of the estimate and the amount of preparation. These studies indicate that the contractor can prepare an accurate cost estimate by spending a large amount of resources; however, in most cases, there is a limit to the resources available for an estimate. It is, therefore, essential to allocate the resources to a number of project contracts in a well-planned manner. Indeed, several studies (Ishii et al., 2013, 2014; Takano et al., 2014) have demonstrated that a contractor's profit can be improved by appropriately allocating the available MH for estimating the costs to each project contract.

Resource allocation has been a subject of active research over the past half a century (Ibaraki and Katoh, 1988; Katoh et al., 2013; Patriksson, 2008), and it has a wide field of application, e.g., R&D project selection (Chen and Zhu, 2011; Heidenberger and Stummer, 1999; Taylor III et al., 1982), marketing budget allocation (Fischer et al., 2011; Soma et al., 2014; Venkatesan and Kumar, 2004), production planning (Bretthauer et al., 2006; Ventura and Klein, 1988; Ziegler, 1982), project budget allocation (Sato and Hirao, 2013), sponsored search auction (Karande et al., 2013; Yang et al., 2012; Zhang et al., 2012) and cloud computing (Beloglazov et al., 2012; Wei et al., 2010; Xiao et al., 2013). Faniran et al. (1999) investigate efficient resource allocation for construction project planning, but they do not take account of competitive bidding. Li and Womer (2006) solve a resource constrained project scheduling problem to formulate a bidding strategy in consideration of due date requirements of projects. However, they ignore the relationship between the accuracy of the cost estimate and the resources allocated to the estimate. To the best of our knowledge, none of the existing studies on resource allocation have addressed the problem of estimating project costs for competitive bidding.

The purpose of the present paper is to establish a novel method of allocating limited resources to the cost estimates of project contracts in a sequential competitive bidding situation. For this purpose, we propose a multi-period resource allocation model, in contrast to the standard single-period models (Ibaraki and Katoh, 1988; Katoh et al., 2013; Patriksson, 2008), to design a long-term resource allocation strategy. This model aims at maximizing the expected profit gained from competitive bidding and is framed as a mixed integer linear programming (MILP) problem by making a piecewise linear approximation of the expected profit function. We demonstrate the effectiveness of our method through numerical experiments examining the characteristics of the optimal resource allocation.

The rest of the paper is organized as follows. In the next section, we first confirm the relationship between the accuracy of the cost estimation and the amount of preparation. We also formulate the expected profit gained from each project contract on the basis of the effort in preparing estimates. Our multi-period resource allocation model is presented in Section 3. The numerical results are reported in Section 4. Finally, conclusions are given in Section 5.

2 Cost Estimating and Competitive Bidding Model

This section explains the correlation between the cost estimation accuracy and the effort to prepare an estimate. Then it describes the competitive bidding model in order to define the expected profit from the project contracts.

2.1 Cost estimate classification matrix

In competitive bidding, contractors first estimate the cost of a project and then decide a bid price on the basis of the estimate. If his/her estimate is relatively high, it will be difficult for him/her to win the contract. Conversely, if his/her estimate is relatively low, s/he is likely to win the contract; however, the project will eventually produce a loss. Therefore, it is critically important for contractors to estimate the project cost accurately.

Table 1, which was created from the cost estimate classification matrix (Christensen and Dysert, 1997), shows the expected accuracy range of project cost estimates. In this table, cost estimates fall into five classes. A Class 5 estimate is the lowest level of project definition and is made with the primary objective of screening or checking feasibility. By contrast, a Class 1 estimate is the closest to a full project definition and is made to check an estimate or submit a bid (see, Christensen and Dysert, 1997). The "Expected Accuracy Range" indicates the degree to which the actual project cost will vary from the estimated cost, and "Preparation Effort" indicates the amount of effort needed to prepare a given estimate. We can see from Table 1 that the estimation accuracy improves as the preparation effort increases. For instance, 0.05% to 0.5% of the project cost is required to make an estimate with an accuracy of −5% to +10%.

Table 1: Expected accuracy range of project cost estimates (Christensen and Dysert, 1997)

	Expected Accuracy Range		
Estimate Class	Lower Limit	Upper Limit	Preparation Effort
Class 5	−100% to −20%	+40% to +200%	0.005%
Class 4	−60% to −15%	+30% to +120%	0.01% to 0.02%
Class 3	−30% to −10%	+20% to +60%	0.015% to 0.05%
Class 2	−15% to −5%	+10% to +30%	0.025% to 0.1%
Class 1	−5%	+10%	0.05% to 0.5%

Note: "Preparation Effort" is expressed as a percentage of the project cost.

2.2 Competitive bidding model

Let us suppose that the contractor plans to bid on contract $i \in \mathcal{I} = \{1, 2, \ldots, I\}$ and estimates its cost with a preparation effort of Class $q \in \mathcal{Q} = \{1, 2, \ldots, Q\}$. The estimated cost $\tilde{E}_{i,q}$ is subject to an unavoidable estimation error, and thus, it is reasonable to treat it as a random variable.

The contractor determines a bid price by putting a markup $m_{i,q}$ on the estimated cost. Therefore, his/her bid price is $(1 + m_{i,q})\tilde{E}_{i,q}$, and if s/he wins the project contract, his/her eventual profit will be $(1 + m_{i,q})\tilde{E}_{i,q} - C_i$, where C_i is the cost of completing the project $i \in \mathcal{I}$. We define $\mathcal{P}_i[b]$, the probability of winning a contract $i \in \mathcal{I}$ as a function of the contractor's bid price, b (see Appendix A for the details). We will set the markup to maximize the expected profit, and thus, the contractor's expected profit obtained from contract $i \in \mathcal{I}$ can be expressed as follows:

$$R_{i,q} = \max_{m_{i,q}} \mathbb{E}\left[\mathcal{P}_i[(1 + m_{i,q})\tilde{E}_{i,q}]((1 + m_{i,q})\tilde{E}_{i,q} - C_i)\right],$$

where $\mathbb{E}[\,\cdot\,]$ is the mathematical expectation. The contractor's expected profit is largely dependent on the estimate class, i.e., the estimation accuracy of the project cost.

To make the expected profit easier to handle, we will use the scenario-based approximation as in Takano et al. (2014). Let $E^{(s)}_{i,q}$ be the estimated cost in scenario $s \in \mathcal{S}$ and P_s be the occurrence probability of scenario $s \in \mathcal{S}$. Accordingly, the expected profit can be rewritten as follows:

$$R_{i,q} = \max_{m_{i,q}} \sum_{s \in \mathcal{S}} P_s \mathcal{P}_i[(1 + m_{i,q})E^{(s)}_{i,q}]((1 + m_{i,q})E^{(s)}_{i,q} - C_i). \tag{1}$$

3 Resource Allocation Model

As mentioned in the previous section, large profits can be expected from project contracts by investing much effort in the cost estimate. It is often the case, however, that contractors are competing on a number of contracts, and the resources available for estimating cost are usually limited in each period. In this section, we consider a multi-period resource allocation model and formulate it as a mixed integer linear programming (MILP) problem.

3.1 Basic optimization model

Let us suppose that the contractor develops a resource allocation strategy for planning periods, $t \in \mathcal{T} = \{1, 2, \ldots, T\}$. More precisely, s/he decides $w_{i,t}$, the preparation effort for estimating the cost of project $i \in \mathcal{I}$ in each period $t \in \mathcal{T}$. It is supposed that the contractor has already started cost estimates of projects $i \in \mathcal{I}_0$ $(\subseteq \mathcal{I})$. Let W^{pre}_i be the preparation effort that has already been invested in $i \in \mathcal{I}$; accordingly, $W^{\text{pre}}_i = 0$ for $i \in \mathcal{I} \setminus \mathcal{I}_0$.

Similarly to Christensen and Dysert (1997), the preparation effort is represented as a percentage of the project cost C_i. Hence, the sum of preparation costs in planning periods $t \in \mathcal{T}$

becomes $C_i \sum_{t\in\mathcal{T}} w_{i,t}$ for each $i \in \mathcal{I}$. In addition, the expected profit $\mathcal{R}_i$ for $i \in \mathcal{I}$ is defined as a function of the preparation effort. Accordingly, the contractor's expected profit gained from contract $i \in \mathcal{I}$ is written as $\mathcal{R}_i(W_i^{\text{pre}} + \sum_{t\in\mathcal{T}} w_{i,t})$. Consequently, our objective becomes maximizing the function,

$$\sum_{i\in\mathcal{I}} \mathcal{R}_i\Big(W_i^{\text{pre}} + \sum_{t\in\mathcal{T}} w_{i,t}\Big) - \sum_{i\in\mathcal{I}} C_i \sum_{t\in\mathcal{T}} w_{i,t}. \tag{2}$$

The budget for cost estimates is usually limited in each period $t \in \mathcal{T}$. Therefore, we take into account the following constraints:

$$\sum_{i\in\mathcal{I}} C_i w_{i,t} \leq B_t \quad (\forall t \in \mathcal{T}), \tag{3}$$

where B_t is the total budget available to cost estimates in period $t \in \mathcal{T}$.

Additionally, let $\mathcal{T}_i$ $(\subseteq \mathcal{T})$ be the periods during which the cost estimation of project i is able to be performed. For instance, it would be useless for contractors to continue to estimate the cost of a project after they have submitted their bid price. Accordingly, we also incorporate the following constraints:

$$\left|\begin{array}{l} w_{i,t} \geq 0 \quad (\forall i \in \mathcal{I}, \forall t \in \mathcal{T}_i), \\ w_{i,t} = 0 \quad (\forall i \in \mathcal{I}, \forall t \notin \mathcal{T}_i). \end{array}\right. \tag{4}$$

Now, our basic optimization model for determining the allocation of efforts, $w_{i,t}$, for estimating the project cost is posed as follows:

$$\left|\begin{array}{ll} \underset{(w_{i,t})}{\text{maximize}} & \sum_{i\in\mathcal{I}} \mathcal{R}_i\Big(W_i^{\text{pre}} + \sum_{t\in\mathcal{T}} w_{i,t}\Big) - \sum_{i\in\mathcal{I}} C_i \sum_{t\in\mathcal{T}} w_{i,t} \\ \text{subject to} & \sum_{i\in\mathcal{I}} C_i w_{i,t} \leq B_t \quad (\forall t \in \mathcal{T}), \\ & w_{i,t} \geq 0 \quad (\forall i \in \mathcal{I}, \forall t \in \mathcal{T}_i), \\ & w_{i,t} = 0 \quad (\forall i \in \mathcal{I}, \forall t \notin \mathcal{T}_i). \end{array}\right. \tag{5}$$

3.2 Piecewise linear approximation

The expected profit function, $\mathcal{R}_i$, is approximated by a piecewise linear function in the following manner. First, we calculate the expected profit $R_{i,q}$ for each contract $i \in \mathcal{I}$ and each estimate class $q \in \mathcal{Q}$ according to equation (1). Next, we make a piecewise linear function as shown in Figure 1, where $W_{i,q}$ is the preparation effort of estimate class $q \in \mathcal{Q}$ for contract $i \in \mathcal{I}$. Specifically, by introducing decision variables $e_{i,q}$ corresponding to the internal division ratio, we approximate the expected profit function, $\mathcal{R}_i$, as follows:

$$\sum_{i\in\mathcal{I}} \mathcal{R}_i\Big(W_i^{\text{pre}} + \sum_{t\in\mathcal{T}} w_{i,t}\Big) \approx \sum_{i\in\mathcal{I}} \sum_{q\in\mathcal{Q}} e_{i,q} R_{i,q} \tag{6}$$

subject to the following constraints:

$$\left| \begin{array}{l} W_i^{\text{pre}} + \sum_{t \in \mathcal{T}} w_{i,t} = \sum_{q \in \mathcal{Q}} e_{i,q} W_{i,q} \quad (\forall i \in \mathcal{I}), \\ \sum_{q \in \mathcal{Q}} e_{i,q} = 1 \quad (\forall i \in \mathcal{I}), \\ e_{i,q} \geq 0 \quad (\forall i \in \mathcal{I}, \forall q \in \mathcal{Q}), \\ \{e_{i,q} \mid q \in \mathcal{Q}\} = \text{SOS2} \quad (\forall i \in \mathcal{I}), \end{array} \right. \tag{7}$$

where the constraint "$\{e_{i,q} \mid q \in \mathcal{Q}\} = \text{SOS2}$" is a special ordered set type two (SOS2) constraint (Beale and Tomlin, 1970). The SOS2 constraint implies that at most two consecutive elements of $e_{i,q}$, $q \in \mathcal{Q}$ can have nonzero values, and it is useful for making piecewise linear approximations of nonlinear functions. Due to its usefulness, this constraint is supported by standard mixed integer programming (MIP) solvers.

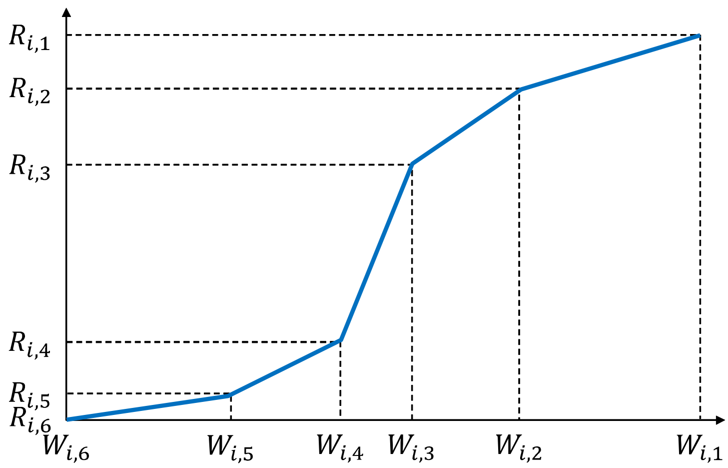

Figure 1: Illustration of piecewise linear function representing the expected profit

3.3 Additional constraints

This subsection provides additional constraints for the basic optimization model (5).

Minimal level of cost estimation accuracy. If the accuracy of the cost estimation is very low, the project costs may be underestimated. In this case, although the contractor is likely to win the contracts, they will cause a large loss. To mitigate the risk of suffering such a large loss, it is effective for the contractor to guarantee a minimal level of cost estimation accuracy. To accomplish this, we introduce 0-1 decision variables,

$$x_i \in \{0, 1\} \quad (\forall i \in \mathcal{I}), \tag{8}$$

which represent whether the contractor will bid or not on each contract $i \in \mathcal{I}$. Next, the following constraints are imposed on the optimization model:

$$\left| \begin{array}{l} W_i^{\mathrm{lo}} x_i \le W_i^{\mathrm{pre}} + \sum_{t \in \mathcal{T}} w_{i,t} \quad (\forall i \in \mathcal{I}), \\ x_i = 1 \quad (\forall i \in \mathcal{I}_0), \\ w_{i,t} \le W_{i,t}^{\mathrm{up}} x_i \quad (\forall i \in \mathcal{I} \setminus \mathcal{I}_0, \forall t \in \mathcal{T}), \end{array} \right. \tag{9}$$

where W_i^{lo} is the lower limit on the preparation effort for contract $i \in \mathcal{I}$, and $W_{i,t}^{\mathrm{up}}$ is the upper limit on it for contract $i \in \mathcal{I}$ in period $t \in \mathcal{T}$. If any preparation effort is invested in contract i, then $x_i = 1$ due to the second and the third lines of constraints (9). It follows from the first line of constraints (9) that the sum of preparation efforts invested in contract i is more than or equal to W_i^{lo}, which guarantees a minimal level of cost estimation accuracy.

Monotonicity of preparation effort. As the date of the competitive bidding approaches, contractors usually increase their effort in making the cost estimate in order to reflect the latest information. It is also undesirable to discontinue a cost estimate temporarily. Let $s(i)$ and $e(i)$ be the first and last periods of estimating the cost of project $i \in \mathcal{I}$, respectively, i.e., $\mathcal{T}_i = \{s(i), s(i)+1, \ldots, e(i)\}$. In view of the above facts, we can place a monotonicity constraint on the preparation effort as follows:

$$W_{i,0} \le w_{i,s(i)} \le w_{i,s(i)+1} \le \cdots \le w_{i,e(i)} \quad (\forall i \in \mathcal{I}), \tag{10}$$

where $W_{i,0}$ is the preparation effort that the contractor invested in the last period, and accordingly, $W_{i,0} = 0$ for $i \in \mathcal{I} \setminus \mathcal{I}_0$.

3.4 MILP formulation

In the basic optimization model (5), the expected profit function is replaced with a piecewise linear function (6) subject to the constraints (7). Moreover, by appending the constraints (8)–(10) to the optimization model, our multi-period resource allocation model for estimating the project costs can be formulated as an MILP problem,

$$
\begin{aligned}
\underset{(x_i),(w_{i,t}),(e_{i,q})}{\text{maximize}} \quad & \sum_{i \in \mathcal{I}} \sum_{q \in \mathcal{Q}} e_{i,q} R_{i,q} - \sum_{i \in \mathcal{I}} C_i \sum_{t \in \mathcal{T}} w_{i,t} \\
\text{subject to} \quad & \sum_{i \in \mathcal{I}} C_i w_{i,t} \le B_t \quad (\forall t \in \mathcal{T}), \\
& w_{i,t} \ge 0 \quad (\forall i \in \mathcal{I}, \forall t \in \mathcal{T}_i), \\
& w_{i,t} = 0 \quad (\forall i \in \mathcal{I}, \forall t \notin \mathcal{T}_i), \\
& W_i^{\text{pre}} + \sum_{t \in \mathcal{T}} w_{i,t} = \sum_{q \in \mathcal{Q}} e_{i,q} W_{i,q} \quad (\forall i \in \mathcal{I}), \\
& \sum_{q \in \mathcal{Q}} e_{i,q} = 1 \quad (\forall i \in \mathcal{I}), \\
& e_{i,q} \ge 0 \quad (\forall i \in \mathcal{I}, \forall q \in \mathcal{Q}), \\
& \{e_{i,q} \mid q \in \mathcal{Q}\} = \text{SOS2} \quad (\forall i \in \mathcal{I}), \\
& x_i \in \{0,1\} \quad (\forall i \in \mathcal{I}), \\
& W_i^{\text{lo}} x_i \le W_i^{\text{pre}} + \sum_{t \in \mathcal{T}} w_{i,t} \quad (\forall i \in \mathcal{I}), \\
& x_i = 1 \quad (\forall i \in \mathcal{I}_0), \\
& w_{i,t} \le W_{i,t}^{\text{up}} x_i \quad (\forall i \in \mathcal{I} \setminus \mathcal{I}_0, \forall t \in \mathcal{T}), \\
& W_{i,0} \le w_{i,s(i)} \le w_{i,s(i)+1} \le \cdots \le w_{i,e(i)} \quad (\forall i \in \mathcal{I}).
\end{aligned}
\tag{11}
$$

4 Numerical Experiments

This section reports numerical results demonstrating the effectiveness of our resource allocation method.

4.1 Problem setting

We supposed that there are 20 project contracts over a planning horizon of nine periods, as shown in Figure 2. It was assumed that projects $i = 1, 2, 6, 9, 12, 15, 18$ are small-scale projects; projects $i = 3, 4, 7, 10, 13, 16, 19$ are medium-scale projects; and projects $i = 5, 8, 11, 14, 17, 20$ are large-scale projects. Note that the contractor has already started cost estimates of projects $i = 1, 2, \ldots, 5$, i.e., $\mathcal{I}_0 := \{1, 2, \ldots, 5\}$. The costs of the other projects can only be estimated during the five periods prior to the date of bidding.

We considered the six estimate classes as shown in Table 2. Note that Class 6 means that the contractor cancels bidding and therefore does not estimate the project cost. Moreover, since the amount of resources required for estimating the cost may vary by industry sector, we considered two cases. The preparation effort of Case A is based on the cost estimate classification matrix (Christensen and Dysert, 1997). In Case B, the contractor needs ten times as much effort as those of Case A to estimate project costs with the same accuracy.

The cost, C_i, of each project scale was 100, 300 and 1,000. It was assumed that the average number of competitors for each contract is five and the mean and standard deviation of each

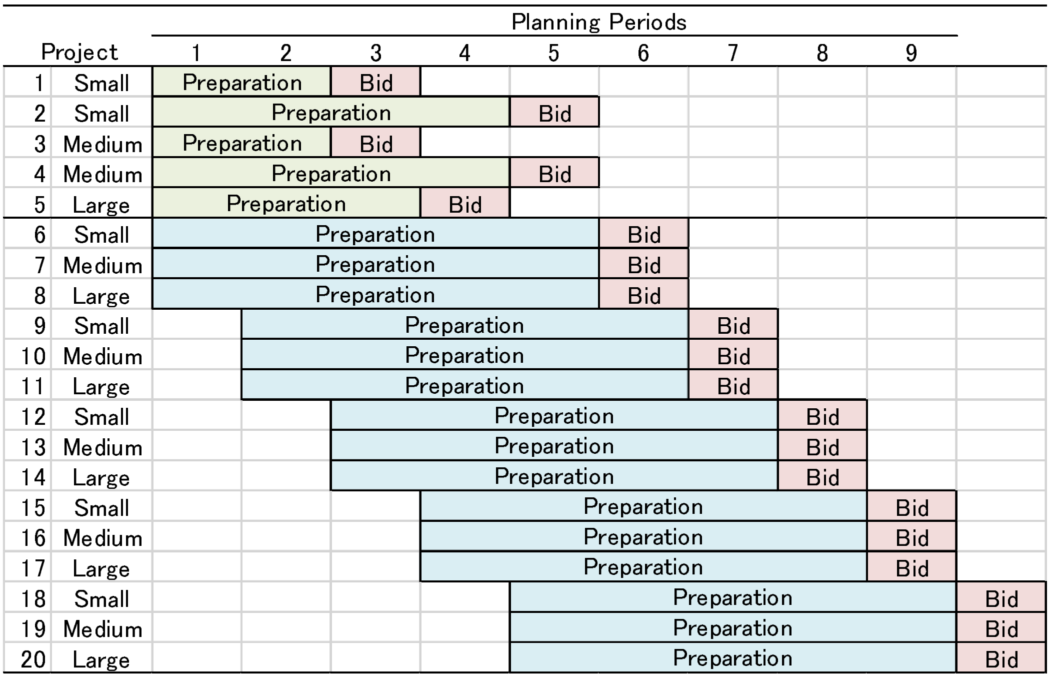

Figure 2: Planning Periods

Table 2: Estimate classes

		Preparation Effort	
Estimate Class	Accuracy Range	Case A	Case B
Class 6	—	0%	0%
Class 5	$[-30\%, 60\%]$	0.015%	0.15%
Class 4	$[-15\%, 30\%]$	0.025%	0.25%
Class 3	$[-10\%, 20\%]$	0.050%	0.50%
Class 2	$[-5\%, 10\%]$	0.100%	1.00%
Class 1	$[-0.5\%, 1.0\%]$	0.500%	5.00%

Note: "Preparation Effort" is expressed as a percentage of the project cost.

competitor's bid price are $1.2 \cdot C_i$ and $0.2 \cdot C_i$. To represent this situation, we set the parameters of the probability of winning as $\kappa_i = 36$, $\theta_i = C_i/30$ and $\lambda_i = 5$ for all $i \in \mathcal{I}$ (see Appendix A for the details). The values of other parameters in problem (11) are shown in Table 3. To investigate how the optimal effort allocation changes depending on the available budget, we analyzed four values of B_t, as shown in Table 3.

4.2 Expected profit from project contracts

Equation (1) was used to calculate the expected profits with respect to each estimate class $q \in \mathcal{Q}$. The estimated costs $E_{i,q}^{(s)}$ of 10,000 scenarios were randomly generated from a beta distribution,

Table 3: Parameter setting

	Case A	Case B
B_t for all $t \in \mathcal{T}$	0.1, 0.2, 0.4, 0.8	1, 2, 4, 8
W_i^{lo} for all $i \in \mathcal{I}$	0.010%	0.10%
$W_{i,t}^{\mathrm{up}}$ for all $i \in \mathcal{I}$ and $t \in \mathcal{T}$	0.500%	5.00%
$W_{i,0}$ for $i = 1, 2, \ldots, 5$	0.005%	0.05%
W_1^{pre}	0.015%	0.15%
W_2^{pre}	0.005%	0.05%
W_3^{pre}	0.015%	0.15%
W_4^{pre}	0.005%	0.05%
W_5^{pre}	0.010%	0.10%

Table 4: Calculated expected profit from each project scale

	Optimal Markup			Expected Profit			Expected Order		
Estimate Class	Small	Medium	Large	Small	Medium	Large	Small	Medium	Large
Class 6	—	—	—	0	0	0	0	0	0
Class 5	0.34	0.34	0.33	0.84	2.50	8.32	7.00	20.56	70.41
Class 4	0.15	0.15	0.15	1.37	4.10	13.73	14.36	43.50	143.49
Class 3	0.12	0.12	0.12	1.64	4.95	16.42	17.37	52.45	173.08
Class 2	0.10	0.10	0.10	1.88	5.65	18.84	20.51	61.47	205.62
Class 1	0.10	0.10	0.10	1.99	5.96	19.86	21.63	64.93	216.56

where the mode of the beta distribution was set to the project cost C_i, and the support of the distribution was set to the corresponding accuracy range shown in Table 2. The occurrence probability, P_s, of each scenario $s \in \mathcal{S}$ was 1/10,000. The optimal markup, $m_{i,q}^*$, was chosen by calculating the expected profit (1) for $m_{i,q} = 0.01, 0.02, \ldots, 0.50$.

Table 4 shows the optimal markup, expected profit, and expected order of each project scale, where the expected order is the average value of the winning bid,

$$\sum_{s \in \mathcal{S}} P_s \mathcal{P}_i[(1 + m_{i,q}^*)E_{i,q}^{(s)}](1 + m_{i,q}^*)E_{i,q}^{(s)}.$$

Figure 3 shows the expected profit functions in Case A, where these functions were created from the expected profits in Table 4. We can see from Figure 3 that they are all monotonically increasing concave functions.

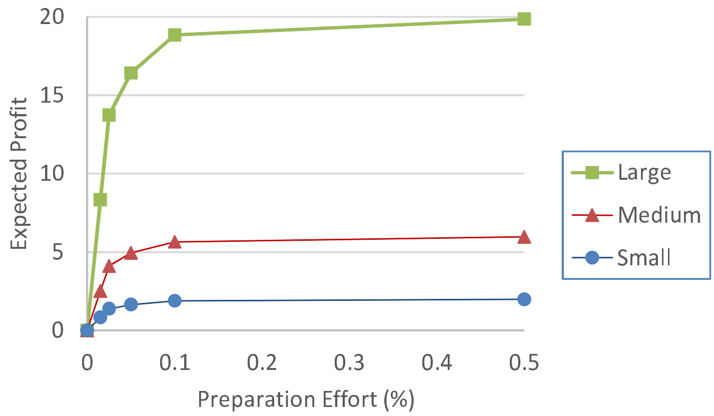

Figure 3: Expected profit function of each project scale in Case A

4.3 Numerical results for Case A

Figures 4–7 show the optimal effort allocation, $w^*_{i,t}$, depending on the total budget in each period in Case A. Here, the column labeled "Pre" is W_i^{pre} for $i \in \mathcal{I}$, and the column labeled "Sum" is $W_i^{\text{pre}} + \sum_{t \in \mathcal{T}} w^*_{i,t}$ for $i \in \mathcal{I}$.

As mentioned above, the contractor has already started estimating the costs of projects $i = 1, 2, \ldots, 5$. Hence, a specific amount of effort must be allocated to these projects on account of the monotonicity constraints (10). For this reason, most of the effort was allocated to projects $i = 1, 2, \ldots, 5$ in Figure 4. As for the other projects $i = 6, 7, \ldots, 20$, effort was allocated only to the large-scale ones, $i = 7, 14, 17, 20$. This suggests that the contractor should prioritize the effort allocation to large-scale projects when the total budget is limited in each period.

Since the total budget was increased in Figure 5, effort was allocated to projects of different scales. Nevertheless, Figure 5 is similar to Figure 4 in that large-scale projects were preferred. More precisely, in Figure 5, an effort of 0.025% was allocated to each large-scale project, whereas less effort was allocated to many small-scale and medium-scale projects.

In Figures 6 and 7, the effort was nearly evenly allocated to all projects. Moreover, we should notice, in contrast to Figures 4 and 5, that the amount of effort allocated to several large-scale projects was less than that allocated to the small-scale and medium-scale projects. In other words, the contractor with a sufficient budget should focus more on evenly allocating effort to all projects than allocating most of the effort to large-scale projects.

4.4 Numerical results for Case B

Figures 8–11 show the optimal effort allocation, $w^*_{i,t}$, depending on the total budget in each period in Case B.

We can see that Figures 8, 9 and 10 are similar to Figures 4, 5 and 6, respectively. However, Figure 11 is distinctly different from Figure 7. Specifically, an effort of 0.1% (Class 2) was

Project		Planning Periods										
		Pre	1	2	3	4	5	6	7	8	9	Sum
1	Small	0.015	0.005	0.005	0	0	0	0	0	0	0	0.025
2	Small	0.005	0.005	0.005	0.005	0.005	0	0	0	0	0	0.025
3	Medium	0.015	0.005	0.005	0	0	0	0	0	0	0	0.025
4	Medium	0.005	0.005	0.005	0.005	0.005	0	0	0	0	0	0.025
5	Large	0.01	0.005	0.005	0.005	0	0	0	0	0	0	0.025
6	Small	0	0	0	0	0	0	0	0	0	0	0
7	Medium	0	0	0	0	0	0	0	0	0	0	0
8	Large	0	0.001	0.001	0.0027	0.0027	0.0027	0	0	0	0	0.01
9	Small	0	0	0	0	0	0	0	0	0	0	0
10	Medium	0	0	0	0	0	0	0	0	0	0	0
11	Large	0	0	0	0	0	0	0	0	0	0	0
12	Small	0	0	0	0	0	0	0	0	0	0	0
13	Medium	0	0	0	0	0	0	0	0	0	0	0
14	Large	0	0	0	0.0003	0.0009	0.0029	0.0029	0.0029	0	0	0.01
15	Small	0	0	0	0	0	0	0	0	0	0	0
16	Medium	0	0	0	0	0	0	0	0	0	0	0
17	Large	0	0	0	0	0.0044	0.0044	0.0044	0.0044	0.0073	0	0.025
18	Small	0	0	0	0	0	0	0	0	0	0	0
19	Medium	0	0	0	0	0	0	0	0	0	0	0
20	Large	0	0	0	0	0	0	0.0027	0.0027	0.0027	0.01	0.018

Figure 4: Optimal effort allocation in Case A ($B_t = 0.1$ for all $t \in \mathcal{T}$)

Project		Planning Periods										
		Pre	1	2	3	4	5	6	7	8	9	Sum
1	Small	0.015	0.005	0.005	0	0	0	0	0	0	0	0.025
2	Small	0.005	0.005	0.005	0.005	0.005	0	0	0	0	0	0.025
3	Medium	0.015	0.005	0.005	0	0	0	0	0	0	0	0.025
4	Medium	0.005	0.005	0.005	0.005	0.005	0	0	0	0	0	0.025
5	Large	0.01	0.005	0.005	0.005	0	0	0	0	0	0	0.025
6	Small	0	0.005	0.005	0.005	0.005	0.005	0	0	0	0	0.025
7	Medium	0	0.005	0.005	0.005	0.005	0.005	0	0	0	0	0.025
8	Large	0	0.005	0.005	0.005	0.005	0.005	0	0	0	0	0.025
9	Small	0	0	0.003	0.003	0.003	0.003	0.003	0	0	0	0.015
10	Medium	0	0	0.0023	0.0023	0.0023	0.0023	0.0023	0	0	0	0.0117
11	Large	0	0	0.003	0.005	0.0057	0.0057	0.0057	0	0	0	0.025
12	Small	0	0	0	0	0	0	0	0.015	0	0	0.015
13	Medium	0	0	0	0	0	0	0	0	0	0	0
14	Large	0	0	0	0	0.0043	0.0063	0.0072	0.0072	0	0	0.025
15	Small	0	0	0	0	0	0	0	0	0.015	0	0.015
16	Medium	0	0	0	0	0	0	0	0	0	0	0
17	Large	0	0	0	0	0	0	0.0062	0.0094	0.0094	0	0.025
18	Small	0	0	0	0	0	0	0	0	0.0075	0.0075	0.015
19	Medium	0	0	0	0	0	0	0	0	0.0075	0.0075	0.015
20	Large	0	0	0	0	0	0	0	0.0019	0.0061	0.017	0.025

Figure 5: Optimal effort allocation in Case A ($B_t = 0.2$ for all $t \in \mathcal{T}$)

allocated to many projects in Figure 7, whereas 0.5% (Class 3) was allocated to all projects in Figure 11. This means that allocating an effort of over 0.5% to each project was not worth it in Case B. We should now recall that the cost estimation was more expensive in Case B than in Case A. Thus, the optimal amount of effort allocated to each contract in Case B was less than that in Case A. Figure 11 indicates that the cost estimates for several projects, e.g., $i = 6, 7, 8$, were finished in a single period. However, some contractors may prefer estimating the cost of each project over multiple periods. In this case, it is only necessary to set the upper limit, $W^{\mathrm{up}}_{i,t}$,

Project		Pre	1	2	3	4	5	6	7	8	9	Sum
		Planning Periods										
1	Small	0.015	0.0175	0.0175	0	0	0	0	0	0	0	0.05
2	Small	0.005	0.0113	0.0113	0.0113	0.0113	0	0	0	0	0	0.05
3	Medium	0.015	0.0175	0.0175	0	0	0	0	0	0	0	0.05
4	Medium	0.005	0.0113	0.0113	0.0113	0.0113	0	0	0	0	0	0.05
5	Large	0.01	0.0133	0.0133	0.0133	0	0	0	0	0	0	0.05
6	Small	0	0.01	0.01	0.01	0.01	0.01	0	0	0	0	0.05
7	Medium	0	0.01	0.01	0.01	0.01	0.01	0	0	0	0	0.05
8	Large	0	0.01	0.01	0.01	0.01	0.01	0	0	0	0	0.05
9	Small	0	0	0	0	0	0.025	0.025	0	0	0	0.05
10	Medium	0	0	0	0	0.0022	0.0089	0.0389	0	0	0	0.05
11	Large	0	0	0.0012	0.0082	0.0092	0.0092	0.0092	0	0	0	0.0368
12	Small	0	0	0	0	0	0	0	0.05	0	0	0.05
13	Medium	0	0	0	0	0.0125	0.0125	0.0125	0.0125	0	0	0.05
14	Large	0	0	0	0	0.0063	0.0063	0.0063	0.0063	0	0	0.025
15	Small	0	0	0	0	0	0	0	0	0.05	0	0.05
16	Medium	0	0	0	0	0	0	0.0167	0.0167	0.0167	0	0.05
17	Large	0	0	0	0	0.0017	0.0017	0.0017	0.01	0.01	0	0.025
18	Small	0	0	0	0	0	0	0	0	0	0.05	0.05
19	Medium	0	0	0	0	0	0	0	0	0	0.05	0.05
20	Large	0	0	0	0	0	0	0	0.01	0.02	0.02	0.05

Figure 6: Optimal effort allocation in Case A ($B_t = 0.4$ for all $t \in \mathcal{T}$)

Project		Pre	1	2	3	4	5	6	7	8	9	Sum
		Planning Periods										
1	Small	0.015	0.0425	0.0425	0	0	0	0	0	0	0	0.1
2	Small	0.005	0.0238	0.0238	0.0238	0.0238	0	0	0	0	0	0.1
3	Medium	0.015	0.0425	0.0425	0	0	0	0	0	0	0	0.1
4	Medium	0.005	0.0238	0.0238	0.0238	0.0238	0	0	0	0	0	0.1
5	Large	0.01	0.0265	0.0265	0.031	0	0	0	0	0	0	0.0939
6	Small	0	0.02	0.02	0.02	0.02	0.02	0	0	0	0	0.1
7	Medium	0	0.02	0.02	0.02	0.02	0.02	0	0	0	0	0.1
8	Large	0	0.019	0.019	0.019	0.019	0.0238	0	0	0	0	0.1
9	Small	0	0	0	0	0	0	0.1	0	0	0	0.1
10	Medium	0	0	0	0	0.0333	0.0333	0.0333	0	0	0	0.1
11	Large	0	0	0	0.0125	0.0125	0.0125	0.0125	0	0	0	0.05
12	Small	0	0	0	0	0	0.0333	0.0333	0.0333	0	0	0.1
13	Medium	0	0	0	0	0.025	0.025	0.025	0.025	0	0	0.1
14	Large	0	0	0	0	0.0115	0.0128	0.0128	0.0128	0	0	0.05
15	Small	0	0	0	0	0.02	0.02	0.02	0.02	0.02	0	0.1
16	Medium	0	0	0	0	0	0	0.0333	0.0333	0.0333	0	0.1
17	Large	0	0	0	0	0	0	0.0118	0.0323	0.0559	0	0.1
18	Small	0	0	0	0	0	0	0	0.0333	0.0333	0.0333	0.1
19	Medium	0	0	0	0	0	0	0	0.0291	0.0291	0.0418	0.1
20	Large	0	0	0	0	0	0	0	0	0	0.0641	0.0641

Figure 7: Optimal effort allocation in Case A ($B_t = 0.8$ for all $t \in \mathcal{T}$)

to a lower value in our resource allocation model.

4.5 Expected profit/order and computation time

Figures 12 and 13 show the contractor's expected profits in Case A and Case B, respectively. As explained above, the cost estimation was more expensive in Case B than in Case A. Hence, the expected profits in Figure 13 were lower than those in Figure 12. These figures also indicate a

Project		Planning Periods Pre	1	2	3	4	5	6	7	8	9	Sum
1	Small	0.15	0.05	0.05	0	0	0	0	0	0	0	0.25
2	Small	0.05	0.05	0.05	0.05	0.05	0	0	0	0	0	0.25
3	Medium	0.15	0.05	0.05	0	0	0	0	0	0	0	0.25
4	Medium	0.05	0.05	0.05	0.05	0.05	0	0	0	0	0	0.25
5	Large	0.1	0.05	0.05	0.05	0	0	0	0	0	0	0.25
6	Small	0	0	0	0	0	0	0	0	0	0	0
7	Medium	0	0	0	0	0	0	0	0	0	0	0
8	Large	0	0.01	0.01	0.03	0.04	0.04	0	0	0	0	0.13
9	Small	0	0	0	0	0	0	0	0	0	0	0
10	Medium	0	0	0	0	0	0	0	0	0	0	0
11	Large	0	0	0	0	0	0	0	0	0	0	0
12	Small	0	0	0	0	0	0	0	0	0	0	0
13	Medium	0	0	0	0	0	0	0	0	0	0	0
14	Large	0	0	0	0	0	0	0	0	0	0	0
15	Small	0	0	0	0	0	0	0	0	0	0	0
16	Medium	0	0	0	0	0	0	0	0	0	0	0
17	Large	0	0	0	0	0.04	0.0525	0.0525	0.0525	0.0525	0	0.25
18	Small	0	0	0	0	0	0	0	0	0	0	0
19	Medium	0	0	0	0	0	0	0	0	0	0	0
20	Large	0	0	0	0	0	0.0075	0.0475	0.0475	0.0475	0.1	0.25

Figure 8: Optimal effort allocation in Case B ($B_t = 1$ for all $t \in \mathcal{T}$)

Project		Planning Periods Pre	1	2	3	4	5	6	7	8	9	Sum
1	Small	0.15	0.05	0.05	0	0	0	0	0	0	0	0.25
2	Small	0.05	0.05	0.05	0.05	0.05	0	0	0	0	0	0.25
3	Medium	0.15	0.05	0.05	0	0	0	0	0	0	0	0.25
4	Medium	0.05	0.05	0.05	0.05	0.05	0	0	0	0	0	0.25
5	Large	0.1	0.05	0.05	0.05	0	0	0	0	0	0	0.25
6	Small	0	0.05	0.05	0.05	0.05	0.05	0	0	0	0	0.25
7	Medium	0	0.05	0.05	0.05	0.05	0.05	0	0	0	0	0.25
8	Large	0	0.05	0.05	0.05	0.05	0.05	0	0	0	0	0.25
9	Small	0	0	0.03	0.03	0.03	0.03	0.03	0	0	0	0.15
10	Medium	0	0	0	0	0	0	0	0	0	0	0
11	Large	0	0	0.037	0.047	0.047	0.0595	0.0595	0	0	0	0.25
12	Small	0	0	0	0	0	0.05	0.05	0.05	0	0	0.15
13	Medium	0	0	0	0	0	0	0	0	0	0	0
14	Large	0	0	0	0.01	0.06	0.06	0.06	0.06	0	0	0.25
15	Small	0	0	0	0	0	0	0	0	0.15	0	0.15
16	Medium	0	0	0	0	0	0	0	0	0.15	0	0.15
17	Large	0	0	0	0	0	0	0.0417	0.1042	0.1042	0	0.25
18	Small	0	0	0	0	0	0	0	0	0	0.15	0.15
19	Medium	0	0	0	0	0	0	0	0	0	0.1167	0.1167
20	Large	0	0	0	0	0	0.0025	0.0308	0.0308	0.0358	0.15	0.25

Figure 9: Optimal effort allocation in Case B ($B_t = 2$ for all $t \in \mathcal{T}$)

relationship between the expected profit and the budget for cost estimates. For instance in Case A, increasing the total budget from 0.1 to 0.2 in each period produced an additional expected profit of about 50. On the other hand, by increasing the total budget from 0.4 to 0.8 in each period, the expected profit improved by only about 15.

Figures 14 and 15 show the expected orders for all project contracts in Case A and Case B. Since the expected order represents the project execution cost after winning the contracts, the

Project		Planning Periods										Sum
		Pre	1	2	3	4	5	6	7	8	9	
1	Small	0.15	0.175	0.175	0	0	0	0	0	0	0	0.5
2	Small	0.05	0.1125	0.1125	0.1125	0.1125	0	0	0	0	0	0.5
3	Medium	0.15	0.175	0.175	0	0	0	0	0	0	0	0.5
4	Medium	0.05	0.1125	0.1125	0.1125	0.1125	0	0	0	0	0	0.5
5	Large	0.1	0.1333	0.1333	0.1333	0	0	0	0	0	0	0.5
6	Small	0	0.1	0.1	0.1	0.1	0.1	0	0	0	0	0.5
7	Medium	0	0.1	0.1	0.1	0.1	0.1	0	0	0	0	0.5
8	Large	0	0.1	0.1	0.1	0.1	0.1	0	0	0	0	0.5
9	Small	0	0	0	0	0	0.25	0.25	0	0	0	0.5
10	Medium	0	0	0	0	0.1222	0.1889	0.1889	0	0	0	0.5
11	Large	0	0	0.0117	0.0317	0.0783	0.0783	0.1683	0	0	0	0.3683
12	Small	0	0	0	0	0	0	0	0.5	0	0	0.5
13	Medium	0	0	0	0	0	0	0	0.5	0	0	0.5
14	Large	0	0	0	0.05	0.05	0.05	0.05	0.05	0	0	0.25
15	Small	0	0	0	0	0	0	0	0	0.5	0	0.5
16	Medium	0	0	0	0	0	0	0	0	0.5	0	0.5
17	Large	0	0	0	0	0.05	0.05	0.05	0.05	0.05	0	0.25
18	Small	0	0	0	0	0	0	0	0	0.25	0.25	0.5
19	Medium	0	0	0	0	0	0	0	0	0	0.5	0.5
20	Large	0	0	0	0	0	0	0.05	0.1	0.125	0.225	0.5

Figure 10: Optimal effort allocation in Case B ($B_t = 4$ for all $t \in \mathcal{T}$)

Project		Planning Periods										Sum
		Pre	1	2	3	4	5	6	7	8	9	
1	Small	0.15	0.05	0.3	0	0	0	0	0	0	0	0.5
2	Small	0.05	0.05	0.05	0.175	0.175	0	0	0	0	0	0.5
3	Medium	0.15	0.05	0.3	0	0	0	0	0	0	0	0.5
4	Medium	0.05	0.05	0.05	0.175	0.175	0	0	0	0	0	0.5
5	Large	0.1	0.05	0.05	0.3	0	0	0	0	0	0	0.5
6	Small	0	0	0	0	0	0.5	0	0	0	0	0.5
7	Medium	0	0	0	0	0	0.5	0	0	0	0	0.5
8	Large	0	0	0	0	0	0.5	0	0	0	0	0.5
9	Small	0	0	0	0	0	0	0.5	0	0	0	0.5
10	Medium	0	0	0	0	0	0.25	0.25	0	0	0	0.5
11	Large	0	0	0	0	0	0.025	0.475	0	0	0	0.5
12	Small	0	0	0	0	0	0	0	0.5	0	0	0.5
13	Medium	0	0	0	0	0	0	0.25	0.25	0	0	0.5
14	Large	0	0	0	0	0	0	0.125	0.375	0	0	0.5
15	Small	0	0	0	0	0	0	0	0	0.5	0	0.5
16	Medium	0	0	0	0	0	0	0	0.25	0.25	0	0.5
17	Large	0	0	0	0	0	0	0	0.225	0.275	0	0.5
18	Small	0	0	0	0	0	0	0	0	0.25	0.25	0.5
19	Medium	0	0	0	0	0	0	0	0	0.25	0.25	0.5
20	Large	0	0	0	0	0	0	0	0	0.25	0.25	0.5

Figure 11: Optimal effort allocation in Case B ($B_t = 8$ for all $t \in \mathcal{T}$)

workload requirements for executing the project can be estimated from these figures.

All computations were conducted on a Windows 7 personal computer with a Core i7 Processor (2.80 GHz) and 8 GB memory. FICO Xpress 7.4[1] was used to solve the optimization problems. The computation time of solving the MILP problem (11) was always less than 0.05 seconds. Thus, even if the number of projects and periods are increased, we expect that our resource

[1] `http://www.fico.com/en`

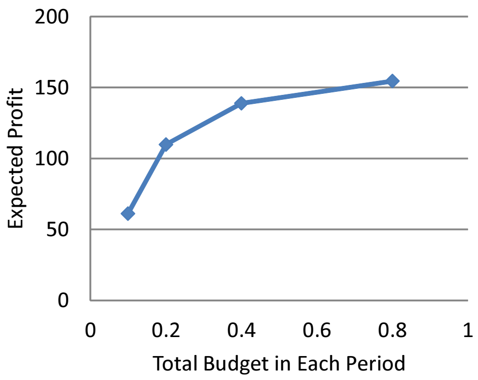

Figure 12: Expected profit in Case A

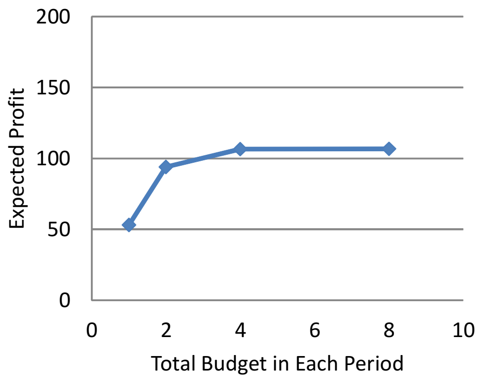

Figure 13: Expected profit in Case B

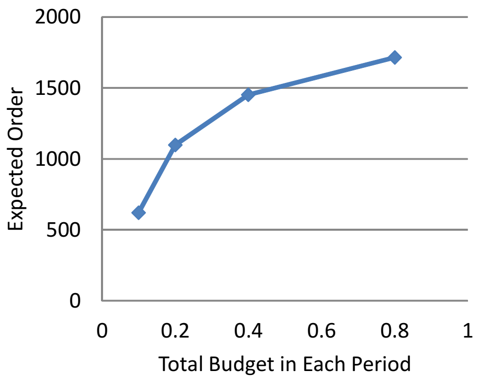

Figure 14: Expected order in Case A

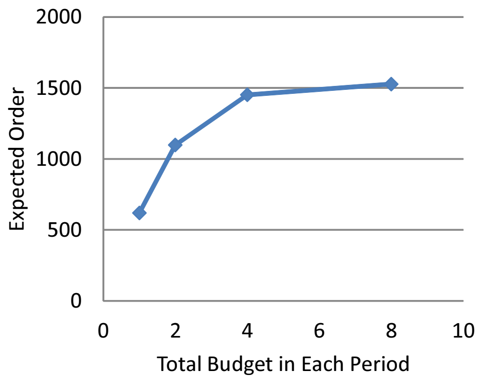

Figure 15: Expected order in Case B

allocation problem can be solved in a reasonable amount of time.

5 Conclusion

This paper described a resource allocation method for estimating the costs of projects in a sequential competitive bidding situation. Moreover, it described numerical experiments showing the validity of the method.

In competitive bidding, it is essential for contractors to estimate project costs accurately. Although previous studies (Christensen and Dysert, 1997; Towler and Sinnott, 2012) indicate a close connection between the accuracy of a cost estimation and resources allocated to it, a resource allocation model for estimating these costs has never been examined. In view of these facts, we developed a framework for appropriately allocating resources to cost estimates, which is the major contribution of this research. Specifically, the expected profits can be calculated from the competitive bidding model. A piecewise linearization technique is used to represent expected profit functions, and the multi-period resource allocation model is posed as a mixed integer linear programming (MILP) problem. This versatile approach can be applied to resource/budget allocation problems in a variety of areas.

The numerical results demonstrated that the resources used to estimate costs should be preferentially allocated to large-scale project contracts when the budget for cost estimates is limited. In contrast, when there is sufficient budget for estimating the cost, it is beneficial to evenly allocate the resources to all contracts to be bid on. Moreover, since our resource allocation model allows one to analyze the relationship between the expected profit and the budgets for the estimates, it will be of practical value.

We may say that this research is of essential importance for generating profits in competitive bidding; however, there are still some issues to be tackled. From a practical viewpoint, the details of project contracts (i.e., date of bidding, project scale, amount of effort necessary for estimating the cost, and so on) are subject to change. Hence, a direction of future research is to incorporate the uncertainty about the details of the project contracts into the resource allocation model. Additionally, although our resource allocation model supports the contractor's decision-making from a long-term perspective, more flexible methods might be needed to determine a resource allocation dynamically in response to the announcement of a new competitive bidding.

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Appendix A Probability of Winning

The function $\mathcal{P}_i$, i.e., the probability of winning a contract i is necessary to calculate the expected profit (1). By following Takano et al. (2014), this appendix describes the formula for computing the probability of winning derived by Friedman (1956), who assumes that the number of competitors follows a Poisson distribution and their bid prices follow identical gamma distributions.

Let us suppose that the bid prices of the competitors for contract i have the same probability density function:

$$\mathcal{F}_i(y) = y^{\kappa_i - 1} \frac{\exp(-y/\theta_i)}{(\kappa_i - 1)!\, \theta_i^{\kappa_i}}, \ y \geq 0, \tag{A.1}$$

where y is the bid price, $\kappa_i\,(\geq 1)$ is a shape parameter, and $\theta_i\,(> 0)$ is a scale parameter. The mean and variance of the gamma distribution (A.1) are $\kappa_i \theta_i$ and $\kappa_i \theta_i^2$, respectively. Moreover, suppose that the number of competitors who bid for contract i has the following probability mass function:

$$\mathcal{G}_i(k) = \frac{\lambda_i^k}{k!} \exp(-\lambda_i), \ k = 0, 1, 2, \ldots, \tag{A.2}$$

where k is the number of competitors who bid for contract i, and λ_i (> 0) is a parameter which represents both the mean and variance of the Poisson distribution (A.2). It then follows from Friedman (1956) that

$$\mathcal{P}_i[b] = \sum_{k=0}^{\infty} \mathcal{G}_i(k) \left(\int_b^{\infty} \mathcal{F}_i(y)\,\mathrm{d}y \right)^k = \exp\left(-\lambda_i \left(1 - \exp\left(-\frac{b}{\theta_i} \right) \sum_{\ell=0}^{\kappa_i - 1} \frac{1}{\ell!} \left(\frac{b}{\theta_i} \right)^{\ell} \right) \right), \qquad \text{(A.3)}$$

where b is a contractor's bid price.

It is often the case that the results of competitive bidding are always announced. So it is possible to estimate the parameters κ_i, θ_i and λ_i by studying previous bidding data of potential competitors.

References

Beale, E. M. L., & Tomlin, J. A. (1970). Special facilities in a general mathematical programming system for non-convex problems using ordered sets of variables. *Proceedings of the 5th International Conference on Operations Research*, 447–454.

Beloglazov, A., Abawajy, J., & Buyya, R. (2012). Energy-aware resource allocation heuristics for efficient management of data centers for cloud computing. *Future Generation Computer Systems, 28*, 755–768.

Bretthauer, K. M., Shetty, B., Syam, S., & Vokurka, R. J. (2006). Production and inventory management under multiple resource constraints. *Mathematical and Computer Modelling, 44*, 85–95.

Chen C. M., & Zhu, J. (2011). Efficient resource allocation via efficiency bootstraps: An application to R&D project budgeting. *Operations Research, 59*, 729–741.

Christensen, P., & Dysert, L. R. (1997). Cost estimate classification system. *AACE International Recommended Practice, 17R-97*. Retrieved from `http://www.anvari.net/Risk Analysis/17r-97.pdf`

Engelbrecht-Wiggans, R. (1980). Auctions and bidding models: A survey. *Management Science, 26*, 119–142.

Faniran, O. O., Love, P. E., & Li, H. (1999). Optimal allocation of construction planning resources. *Journal of Construction Engineering and Management, 125*, 311–319.

Fischer, M., Albers, S., Wagner, N., & Frie, M. (2011). Dynamic marketing budget allocation across countries, products, and marketing activities. *Marketing Science, 30*, 568–585.

Friedman, L. (1956). A competitive-bidding strategy. *Operations Research, 4*, 104–112.

Heidenberger, K., & Stummer, C. (1999). Research and development project selection and resource allocation: A review of quantitative modelling approaches. *International Journal of Management Reviews, 1*, 197–224.

Ibaraki, T., & Katoh, N. (1988). *Resource Allocation Problems: Algorithmic Approaches.* Cambridge: MIT Press.

Ishii, N., Takano, Y., & Muraki, M. (2013). A two-step bidding price decision algorithm under limited man-hours in EPC projects. *Proceedings of the 3rd International Conference on Simulation and Modeling Methodologies, Technologies and Applications (SIMULTECH)*, 393–403.

Ishii, N., Takano, Y., & Muraki, M. (2014). An order acceptance strategy under limited engineering man-hours for cost estimation in engineering-procurement-construction projects. *International Journal of Project Management, 32*, 519–528.

Karande, C., Mehta, A., & Srikant, R. (2013). Optimizing budget constrained spend in search advertising. *Proceedings of the 6th ACM International Conference on Web Search and Data Mining (WSDM'13)*, 697–706.

Katoh, N., Shioura, A., & Ibaraki, T. (2013). Resource allocation problems. In P. Pardalos, D.-Z. Du, & R. L. Graham (Eds.), *Handbook of Combinatorial Optimization* (2nd Ed., pp. 2897–2988). New York: Springer.

King, M., & Mercer, A. (1988). Recurrent competitive bidding. *European Journal of Operational Research, 33*, 2–16.

King, M., & Mercer, A. (1990). The optimum markup when bidding with uncertain costs. *European Journal of Operational Research, 47*, 348–363.

Li, H., & Womer, K. (2006). Project scheduling in decision-theoretic competitive bidding. *2006 IEEE Congress on Evolutionary Computation*, 3042–3049.

Naert, P. A., & Weverbergh, M. (1978). Cost uncertainty in competitive bidding models. *Journal of the Operational Research Society, 29*, 361–372.

Patriksson, M. (2008). A survey on the continuous nonlinear resource allocation problem. *European Journal of Operational Research, 185*, 1–46.

Rothkopf, M. H., & Harstad, R. M. (1994). Modeling competitive bidding: A critical essay. *Management Science, 40*, 364–384.

Sato, T., & Hirao, M. (2013). Optimum budget allocation method for projects with critical risks. *International Journal of Project Management, 31*, 126–135.

Soma, T., Kakimura, N., Inaba, K., & Kawarabayashi, K. (2014). Optimal budget allocation: Theoretical guarantee and efficient algorithm. *Proceedings of the 31st International Conference on Machine Learning (ICML'14)*, 351–359.

Stark, R. M., & Rothkopf, M. H. (1979). Competitive bidding: A comprehensive bibliography. *Operations Research, 27*, 364–390.

Takano, Y., Ishii, N., & Muraki, M. (2014). A sequential competitive bidding strategy considering inaccurate cost estimates. *OMEGA, The International Journal of Management Science, 42*, 132–140.

Taylor III, B. W., Moore, L. J., & Clayton, E. R. (1982). R&D project selection and manpower allocation with integer nonlinear goal programming. *Management Science, 28*, 1149–1158.

Towler, G., & Sinnott, R. K. (2012). *Chemical Engineering Design* (2nd Ed.). Waltham: Butterworth-Heinemann.

Venkatesan, R., & Kumar, V. (2004). A customer lifetime value framework for customer selection and resource allocation strategy. *Journal of Marketing, 68*, 106–125.

Ventura, J. A., & Klein, C. M. (1988). A note on multi-item inventory systems with limited capacity. *Operations Research Letters, 7*, 71–75.

Wei, G., Vasilakos, A. V., Zheng, Y., & Xiong, N. (2010). A game-theoretic method of fair resource allocation for cloud computing services. *The Journal of Supercomputing, 54*, 252–269.

Xiao, Z., Song, W., & Chen, Q. (2013). Dynamic resource allocation using virtual machines for cloud computing environment. *IEEE Transactions on Parallel and Distributed Systems, 24*, 1107–1117.

Yang, Y., Zhang, J., Qin, R., Li, J., Wang, F. Y., & Qi, W. (2012). A budget optimization framework for search advertisements across markets. *IEEE Transactions on Systems, Man and Cybernetics — Part A: Systems and Humans, 42*, 1141–1151.

Zhang, W., Zhang, Y., Gao, B., Yu, Y., Yuan, K., & Liu, T. Y. (2012). Joint optimization of bid and budget allocation in sponsored search. *Proceedings of the 18th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (KDD'12)*, 1177–1185.

Ziegler, H. (1982). Solving certain singly constrained convex optimization problems in production planning. *Operations Research Letters, 1*, 246–252.