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A Sequential Competitive Bidding Strategy Considering Inaccurate Cost Estimates

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Abstract

This paper develops a stochastic dynamic programming model for establishing an optimal sequential bidding strategy in a competitive bidding situation. In competitive bidding, a contractor usually sets the bid price of each contract by putting a markup on the estimated cost, and consequently, the bid price is affected by a cost estimation error. We take a scenario-based approach to determine the optimal markup in consideration of the effect of inaccurate cost estimates. We also introduce a value-at-risk constraint to mitigate the risk of suffering a large loss. Numerical results show that our model increases the average profit and reduces the profit volatility risk.

Keywords: Bidding, Dynamic programming, Sequential bidding strategy, Value-at-risk

1 Introduction

Competitive bidding has been widely used to choose contractors who will execute a contract at the lowest cost. In this process, a contractor, who has received an invitation from a potential client, estimates the cost for completing the contract. S/he then sets a bid price by putting a markup on the estimated cost. If his/her bid is the lowest among the competitors', s/he wins the contract. The true cost (and actual profit) of the contract can only be determined after completion of the corresponding project. Hence, an accurate bidding strategy is crucial to secure a profit from such contracts.

The origin of research on optimal bidding strategies dates back to the early work of Friedman (1956). A considerable number of studies over the past half a century have since dealt with competitive bidding strategies (see, e.g., Engelbrecht-Wiggans, 1980; King and Mercer, 1988; Rothkopf and Harstad, 1994; Stark and Rothkopf, 1979, for detailed references). Friedman (1956) determined the optimal bid price under uncertainty about the true cost. This makes sense because the true cost is uncertain when the winning bid is determined. However, Friedman (1956) ignored the fact that the bid price itself is affected by the inaccuracies in the estimated cost. If the cost is underestimated by a contractor, his/her bid price will be relatively low. In this case, the contractor can win the contract; however, s/he stands to suffer from cost overruns. On the other hand, if his/her estimated cost is substantially higher than the true cost, s/he will probably fail to win the contract because the bid will be relatively higher than the others. Making accurate cost estimates of IT projects, for instance, has proven extremely difficult even when novel estimation methods such as artificial neural networks are used (see Berlin et al., 2009, and the references therein), and thus, there is no disregarding the effects of an inaccurate cost estimate on the actual profit. However, the uncertainty about the estimated cost complicates the formula for determining the probability of winning and makes the model difficult to handle.

Naert and Weverbergh (1978) were the first to propose a solution for finding the optimal markup in view of the effect of cost uncertainty on the bid price. King and Mercer (1990) obtained analytical solutions for different distributions of the estimated cost, for different versions of the expected profit, and for different formula for the probability of winning. Ishii et al. (2011) determined not only the optimal bid prices but also the optimal allocation of manpower in the cost estimation of each contract. Although these studies (Ishii et al., 2011; King and Mercer, 1990; Naert and Weverbergh, 1978) considered the estimated cost to be uncertain when determining the optimal bid price, they discussed only “one-shot” bidding.

A bidder’s strategy and result may affect his/her subsequent bidding strategy. Let us assume that a contractor bids for long-term projects. In this case, winning a large number of contracts will inevitably lead to man-hour shortfall sometime in the future. Accordingly, s/he should select projects and bid for them from a long-term standpoint. Since there may be such a complementary relationship among projects, a sequential bidding strategy has to be drawn up. Because of these factors, several articles have dealt with “sequential” bidding strategies for competitive bidding (Attanasi, 1974; Knode and Swanson, 1978; Kortanek et al., 1973; Li and Womer, 2006; Oren and Rothkopf, 1975; Stark and Mayer Jr., 1971) and combinatorial auctions (Boutilier et al., 1999a,b; Hattori et al., 2001). To the best of our knowledge, however, none of the existing studies on sequential bidding strategies have taken into account uncertainty about the estimated cost because the corresponding complex dynamic programming problem is intractable.

This paper develops a stochastic dynamic programming model for establishing an optimal sequential bidding strategy in a competitive bidding situation. Our dynamic programming model is somewhat similar to the models developed by Knode and Swanson (1978). However, as mentioned above, our model differs from theirs (Knode and Swanson, 1978) by considering not the true cost but the estimated cost with uncertainty. We take a scenario-based approach, which is also different from the ones taken in the previous articles (Ishii et al., 2011; King and Mercer, 1990; Naert and Weverbergh, 1978), to determine the optimal markup in consideration of an inaccurate cost estimation. This approach has an advantage in that there is no need to make an assumption about a probability distribution of the cost estimation error, unlike the previous articles (Ishii et al., 2011; King and Mercer, 1990; Naert and Weverbergh, 1978). Furthermore, we employ a financial risk measure, value-at-risk (VaR, see, e.g., Duffie and Pan, 1997), to reduce the risk of suffering a large loss. Most of the existing bidding models have focused on maximizing the expected profit, while very few have attempted to mitigate the risk from inaccurate cost estimates.

We conducted numerical simulations to evaluate the performance of our bidding strategy. These simulations also illustrate value of this paper because the previous articles (Attanasi, 1974; Knode and Swanson, 1978; Kortanek et al., 1973; Li and Womer, 2006; Oren and Rothkopf, 1975; Stark and Mayer Jr., 1971) on sequential competitive bidding did not provide detailed simulation results. The numerical results show that our model increases the average profit and reduces the profit volatility risk by taking into account uncertainty about the estimated cost.

The rest of the paper is organized as follows: In Section 2, we present a stochastic dynamic programming model and clarify the difference between our model and the model based on Friedman (1956). In Section 3, we explain how to define the probability of winning contracts. Numerical results are given in Section 4, and conclusions are drawn in Section 5.

2 Stochastic Dynamic Programming Model

2.1 Preliminaries

The terminology and notation used in this paper are as follows:

Index Sets

$\mathcal{I} := \{1, 2, \ldots, I\}$; index set of project contracts

$\mathcal{S} := \{1, 2, \ldots, S\}$; index set of scenarios of the estimated cost

$\mathcal{T} := \{1, 2, \ldots, T\}$; index set of planning time periods

Decision Variables

m_i : markup of project contract i $(i \in \mathcal{I})$

Random Variables

$\tilde{E}_i$: estimated cost of project i $(i \in \mathcal{I})$

Given Constants

C_i : true cost of project i $(i \in \mathcal{I})$

E_{is} : estimated cost of project i under scenario s $(i \in \mathcal{I},\ s \in \mathcal{S})$

P_s : occurrence probability of scenario s $(s \in \mathcal{S})$

W_{it} : man-hours necessary for executing project i in period t $(i \in \mathcal{I},\ t \in \mathcal{T})$

M_t : available man-hours in period t $(t \in \mathcal{T})$

L_i (U_i) : lower (upper) limit on the markup $(i \in \mathcal{I})$

γ_t : outsourcing cost per a unit of man-hour in period t $(t \in \mathcal{T})$

$\beta\,(\in (0,1))$: confidence level of the VaR

α_i : upper limit of the VaR of project contract i $(i \in \mathcal{I})$

Functions

$\mathcal{P}_i[b]$: probability of winning contract i when the bid price is b $(i \in \mathcal{I})$

$\mu(\tilde{x})$: mean of a random variable $\tilde{x}$

$\sigma(\tilde{x})$: standard deviation of a random variable $\tilde{x}$

2.2 Process of competitive bidding

We shall deal with competitive bidding for long-term projects. Figure 1 shows that we (the contractor) have already received two project contracts and that there are ten project contracts on which we can bid. It is assumed that bids take place in order. That is, bidding for project contract $i+1$ takes place after bidding opens on contract i. Although it may well be that several bids take place simultaneously, we shall not consider such a situation for the sake of simplicity.

To participate in the bidding for contract i, we begin by estimating the cost of project i; however, the estimated cost $\tilde{E}_i$ is subject to unavoidable estimation errors. Next, we determine the markup, m_i, on the estimated cost $\tilde{E}_i$. Consequently, we offer a bid price $(1 + m_i)\tilde{E}_i$ for project contract i. If the bid price is the lowest among the competitors', we will win project

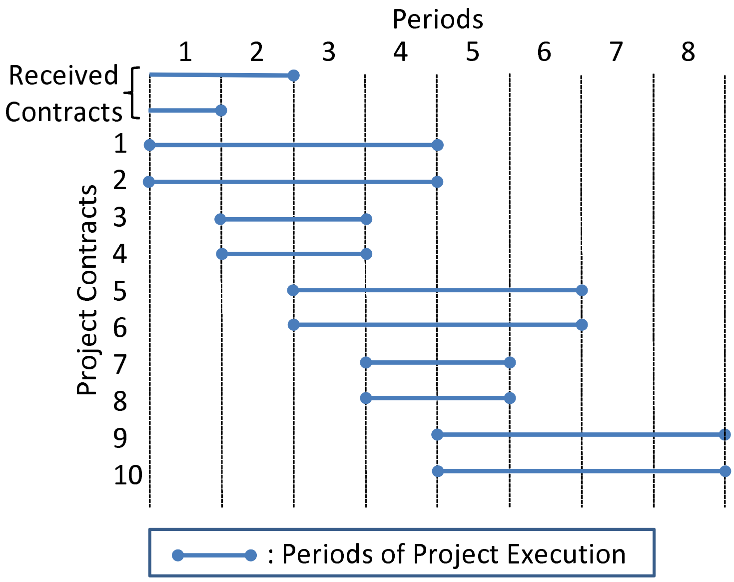


Figure 1: Project Contracts

contract i. After that, we will bid for the next project contract $i+1$.

Since we assume long-term projects, they will each span multiple periods before they are completed. Figure 1 shows that project 1 continues from period 1 to period 4. Also, each project execution requires a certain number of man-hours in each period. Specifically, it is supposed that W_{it} man-hours will be needed for executing a project i within the prescribed period t. It is also supposed that there is a restriction on the available man-hours in each period; the available man-hours in period t is M_t. If the required man-hours in period t exceeds M_t, we will have to outsource to make up for the lack, which costs γ_t per unit man-hour.

2.3 Formulation

A contractor wins a project contract i with a probability $\mathcal{P}_i[(1+m_i)\tilde{E}_i]$. Note that the probability of winning, $\mathcal{P}_i[(1+m_i)\tilde{E}_i]$, is a function of the random estimated cost $\tilde{E}_i$; therefore, whether s/he can win the contract or not depends largely on the cost estimation. If the contractor wins the contract i, s/he will gain a profit $(1+m_i)\tilde{E}_i - C_i$ from the corresponding project i. Therefore, the expected profit from contract i is expressed as follows:

$$\mathbb{E}\left[\mathcal{P}_i[(1+m_i)\tilde{E}_i]((1+m_i)\tilde{E}_i - C_i)\right], \tag{1}$$

where $\mathbb{E}[\,\cdot\,]$ is the mathematical expectation.

Remark 1 (Friedman's Model (Friedman, 1956)). In contrast to this paper, the true cost was random and the estimated cost was given in Friedman (1956). By following Friedman (1956), we can let $\tilde{C}_i$ be the random true cost of project i and E_i be the given estimated cost of project i. Then the expected profit from a project i is expressed as

$$\mathbb{E}\left[\mathcal{P}_i[(1+m_i)E_i]((1+m_i)E_i - \tilde{C}_i)\right] = \mathcal{P}_i[(1+m_i)E_i]((1+m_i)E_i - \mu(\tilde{C}_i)). \tag{2}$$

We call (2) Friedman's model in this paper. Note that the expected profit (2) is completely unaffected by the cost uncertainty, and accordingly, it is clear that it is easier to handle Friedman's model (2) than to handle model (1). □

In what follows, it is assumed that the estimated cost $\tilde{E}_i$ follows a normal distribution with mean $\mu(\tilde{E}_i)$ and variance $\sigma(\tilde{E}_i)^2$. These parameters, i.e., $\mu(\tilde{E}_i)$ and $\sigma(\tilde{E}_i)$, can be estimated from the data of previous project contracts won through competitive bidding. We randomly generate scenarios of the estimated cost by drawing samples from the normal distribution $\mathbf{N}(\mu(\tilde{E}_i), \sigma(\tilde{E}_i)^2)$. We denote by E_{is} the estimated cost of project i under scenario s, and by P_s the occurrence probability of scenario s.

To make model (1) easier to handle, we can rewrite it as follows:

$$\sum_{s\in\mathcal{S}} P_s \mathcal{P}_i[(1+m_i)E_{is}]((1+m_i)E_{is} - C_i). \tag{3}$$

Note that our model (3) is an extension of Friedman's model (2) because (2) is a special case of (3), by setting

$$S := 1,\ P_1 := 1,\ E_{i1} := E_i \text{ for } i \in \mathcal{I},\ \text{and } C_i := \mu(\tilde{C}_i) \text{ for } i \in \mathcal{I}, \tag{4}$$

in (3).

When a set of contracts $\mathcal{J}(\subseteq \mathcal{I})$ is won, the contractor may have to outsource man-hours in the amount, $\max\{\sum_{i\in\mathcal{J}} W_{it} - M_t,\ 0\}$, during each period $t \in \mathcal{T}$. Consequently, the total outsourcing cost is

$$\sum_{t\in\mathcal{T}} \gamma_t \max\left\{\sum_{i\in\mathcal{J}} W_{it} - M_t,\ 0\right\}. \tag{5}$$

Furthermore, we impose a value-at-risk (VaR, see, e.g., Duffie and Pan, 1997) constraint to reduce the risk of suffering a large loss. β-VaR is defined as the β-quantile of a random loss (see

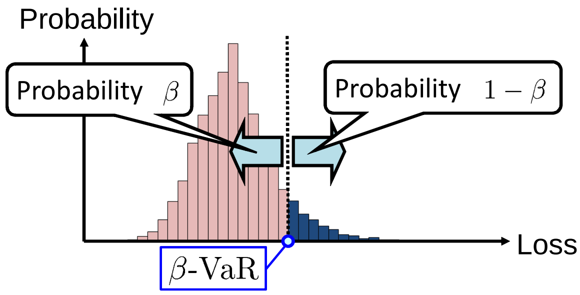


Figure 2: Value-at-Risk

Figure 2), where the confidence level β is usually set to 0.9, 0.95, and so on. Considering that the random loss is $C_i - (1+m_i)\tilde{E}_i$, the VaR on each project contract $i \in \mathcal{I}$ is expressed by

$$\min\{\, v \mid \text{Prob}\{C_i - (1+m_i)\tilde{E}_i \leq v\} \geq \beta \,\}, \tag{6}$$

where $\text{Prob}\{A\}$ denotes the probability of an event A. We shall impose an upper limit α_i on the VaR (6) and refer to it as the VaR constraint. When the upper limit, α_i, is 0, the VaR constraint implies that the probability of project i being in the red is no more than $1-\beta$.

Since the estimated cost, $\tilde{E}_i$, follows a normal distribution with mean $\mu(\tilde{E}_i)$ and variance $\sigma(\tilde{E}_i)^2$, we can rewrite the VaR constraint as

$$C_i - (1+m_i)\left(\mu(\tilde{E}_i) - \mathcal{Z}^{-1}(\beta)\sigma(\tilde{E}_i)\right) \leq \alpha_i, \tag{7}$$

where $\mathcal{Z}$ is the cumulative distribution function of the standard normal distribution. Assuming that $\mu(\tilde{E}_i) - \mathcal{Z}^{-1}(\beta)\sigma(\tilde{E}_i) > 0$, it follows from (7) that the VaR constraint can be reduced to a lower limit constraint on the markup, i.e., $L_i \leq m_i$, where

$$L_i := \frac{C_i - \alpha_i}{\mu(\tilde{E}_i) - \mathcal{Z}^{-1}(\beta)\sigma(\tilde{E}_i)} - 1. \tag{8}$$

It is necessary to assume that the estimated cost, $\tilde{E}_i$, follows normal distribution in order to equivalently convert the VaR constraint into the lower-limit constraint with (8). However, we should notice that this assumption is unnecessary if we have only to maximize the expected profit (3). This is because the scenarios of the estimated cost, E_{is}, can be generated from any probability distribution.

2.4 Dynamic programming recursion

Our objective is to maximize the expected total profit, which is the sum of the expected profits (3) for all $i \in \mathcal{I}$ minus the expected value of the outsourcing cost (5).

Let us suppose that the set of contracts $\mathcal{J}$ ($\subseteq \{1, 2, \ldots, i-1\}$) has already been received. If we win contract i, the expected profit from the contracts $i, i+1, \ldots, I$ is

$$\sum_{s \in \mathcal{S}} P_s \left((1 + m_i) E_{is} - C_i + \mathcal{V}_{i+1}(\mathcal{J} \cup \{i\}) \right),$$

where $\mathcal{V}_{i+1}(\mathcal{J} \cup \{i\})$ represents the expected profit from the contracts $i+1, i+2, \ldots, I$ for which we set the optimal bid prices. Conversely, if we lose contract i, $\mathcal{V}_{i+1}(\mathcal{J})$ represents the expected profit from the contracts $i+1, i+2, \ldots, I$ for which we set the optimal bid prices.

We win the contract i with probability $\mathcal{P}_i[(1 + m_i)E_{is}]$ and lose contract i with probability $1 - \mathcal{P}_i[(1 + m_i)E_{is}]$ under scenario s. Consequently,

$$\begin{aligned} \mathcal{Q}_i(\, m_i \mid \mathcal{J}\,) := & \sum_{s \in \mathcal{S}} P_s \mathcal{P}_i[(1 + m_i)E_{is}] \left((1 + m_i) E_{is} - C_i + \mathcal{V}_{i+1}(\mathcal{J} \cup \{i\}) \right) \\ & + \sum_{s \in \mathcal{S}} P_s (1 - \mathcal{P}_i[(1 + m_i)E_{is})] \mathcal{V}_{i+1}(\mathcal{J}),\ i \in \mathcal{I},\ \mathcal{J} \subseteq \{1, 2, \ldots, i-1\} \end{aligned} \tag{9}$$

is the objective function to be maximized by determining the markup m_i of contract i for the received set of contracts $\mathcal{J}$ ($\subseteq \{1, 2, \ldots, i-1\}$). The optimal objective value is

$$\mathcal{V}_i(\mathcal{J}) := \max_{m_i} \left\{ \mathcal{Q}_i(\, m_i \mid \mathcal{J}\,) \mid L_i \leq m_i \leq U_i \right\},\ i \in \mathcal{I},\ \mathcal{J} \subseteq \{1, 2, \ldots, i-1\}. \tag{10}$$

In addition, $\mathcal{V}_{I+1}(\mathcal{J})$ is set to the negative of the necessary outsourcing cost of projects $\mathcal{J}$ ($\subseteq \mathcal{I}$) as follows:

$$\mathcal{V}_{I+1}(\mathcal{J}) := -\sum_{t \in \mathcal{T}} \gamma_t \max \left\{ \sum_{i \in \mathcal{J}} W_{it} - M_t,\ 0 \right\},\ \mathcal{J} \subseteq \mathcal{I}. \tag{11}$$

To develop an optimal bidding strategy, we solve the recursion equations (10) backward. That is, we first calculate $\mathcal{V}_I(\mathcal{J})$ for all $\mathcal{J} \subseteq \{1, 2, \ldots, I-1\}$ from (9) and (11). Next, we calculate $\mathcal{V}_{I-1}(\mathcal{J})$ for all $\mathcal{J} \subseteq \{1, 2, \ldots, I-2\}$ and calculate $\mathcal{V}_{I-2}(\mathcal{J})$ for all $\mathcal{J} \subseteq \{1, 2, \ldots, I-3\}$ in a similar manner. Finally, we obtain $\mathcal{V}_1(\emptyset)$, which means that the optimal bidding strategy is established.

Assuming that we bid for long-term projects, $\mathcal{V}_{I+1}(\mathcal{J})$ can be defined by (11) on the basis of the outsourcing cost. However, our dynamic programming model is also useful in the following situation. For instance, if we win the set of project contracts $\mathcal{J}'$ that require the same machinery and technology, it has the potential to cut the costs of completing those projects. Such synergistic effect or complementary relationship among projects should be integrated into a sequential bidding strategy. To do this, in this instance, we should set $\mathcal{V}_{I+1}(\mathcal{J}')$ to the cost reduction inherent for a set of similar projects $\mathcal{J}'$.

3 Probability of Winning Contracts

The only task left for us is how to define the function $\mathcal{P}_i$ which is the probability of winning a contract i. Since a number of competitors participate in the bidding, the uncertain elements are the number of competitors who will bid and their bid prices. Assuming that the number of competitors follows a Poisson distribution and that competitors' bid prices follow identical gamma distributions, Friedman (1956) derived a formula for the probability of winning, $\mathcal{P}_i$. Although several previous studies have proposed the use of other distributions (see, e.g., Engelbrecht-Wiggans, 1980; King and Mercer, 1988; Rothkopf and Harstad, 1994; Stark and Rothkopf, 1979), Friedman's probability of winning is reasonably designed and easy to handle. Accordingly, we shall use it to present the numerical results in the next section.

Remark 2 (Friedman's Probability of Winning (Friedman, 1956)). Suppose that the bid prices of the competitors for contract i have the same probability density function:

$$\mathcal{F}_i(y) = y^{\kappa_i - 1} \frac{\exp(-y/\theta_i)}{(\kappa_i - 1)!\, \theta_i^{\kappa_i}}, \ y \geq 0, \tag{12}$$

where y is the bid price, $\kappa_i\ (\geq 1)$ is a shape parameter, and $\theta_i\ (> 0)$ is a scale parameter. The mean and variance of the Gamma distribution (12) are $\kappa_i\theta_i$ and $\kappa_i\theta_i^2$, respectively. Moreover, suppose that the number of competitors who bid for contract i has the following probability mass function:

$$\mathcal{G}_i(k) = \frac{\lambda_i^k}{k!} \exp(-\lambda_i), \ k = 0, 1, 2, \ldots, \tag{13}$$

where k is the number of competitors who bid for contract i, and $\lambda_i\ (> 0)$ is a parameter which represents both the mean and variance of the Poisson distribution (13). It then follows from Friedman (1956) that

$$\mathcal{P}_i[b] := \sum_{k=0}^{\infty} \mathcal{G}_i(k) \left(\int_b^{\infty} \mathcal{F}_i(y)\, \mathrm{d}y \right)^k = \exp\left(-\lambda_i \left(1 - \exp\left(-\frac{b}{\theta_i} \right) \sum_{\ell=0}^{\kappa_i - 1} \frac{1}{\ell!} \left(\frac{b}{\theta_i} \right)^{\ell} \right) \right), \tag{14}$$

where b is a contractor's bid price. □

It is often the case that the results of competitive bidding are always announced. So it is possible to estimate the parameters κ_i, θ_i and λ_i by studying previous bidding data of potential competitors.

4 Numerical Experiments

In this section, we show numerical results to demonstrate the effectiveness of our stochastic dynamic programming model. All computations were conducted on a Windows 7 personal computer with a Core i5 Processor (2.40 GHz) and 4GB memory. MATLAB (R2011b) and a MATLAB optimization toolbox, *fmincon*, were used to implement stochastic dynamic programming recursion.

4.1 Problem setting

We made the problem setting relatively simple in order for readers to grasp the essentials of the numerical results easily. It is supposed that a contractor bids for ten project contracts over a planning horizon of eight periods, i.e., $I := 10$ and $T := 8$, as shown in Figure 1. The true cost, C_i was set to 1.0 for all projects $i \in \mathcal{I}$. The man-hours necessary for executing a project in each period were set as

$$(W_{it}) = \begin{pmatrix} 3 & 3 & 3 & 3 & 0 & 0 & 0 & 0 \\ 3 & 3 & 3 & 3 & 0 & 0 & 0 & 0 \\ 0 & 6 & 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 6 & 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 3 & 3 & 3 & 0 & 0 \\ 0 & 0 & 3 & 3 & 3 & 3 & 0 & 0 \\ 0 & 0 & 0 & 6 & 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 & 3 & 3 & 3 & 3 \end{pmatrix},$$

and the available man-hours in each period were set as $M_1 := 3$, $M_2 := 9$ and $M_t := 12$ for $t = 3, 4, \dots, 8$. In addition, the outsourcing cost, γ_t, was set to 0.1 for all periods $t \in \mathcal{T}$. Since this outsourcing cost is very high, a contractor normally does not bid for contracts that will exceed his/her available man-hours.

We supposed that the estimated cost was equal to the true cost on average; specifically, $\mu(\tilde{E}_i)$ was set to 1.0 for all projects $i \in \mathcal{I}$. Meanwhile, a contractor should be able to adjust the estimation accuracy of the project costs to some extent by changing the manpower to be allocated to each project. Thus to examine the effects of adjusting the cost estimation accuracy, we considered two settings for the standard deviation of the estimated cost:

- Stdev(0.1all): $\sigma(\tilde{E}_i) := 0.1$ for all $i \in \mathcal{I}$, and
- Stdev(0.12&0.08): $\sigma(\tilde{E}_i) := 0.12$ for $i = 1, 3, 5, 7, 9$ and $\sigma(\tilde{E}_i) := 0.08$ for $i = 2, 4, 6, 8, 10$.

The cost estimation accuracy of projects $i = 2, 4, 6, 8, 10$ is higher than that of other projects in Stdev(0.12&0.08), whereas the cost estimation accuracy of all projects is the same in Stdev(0.1all). For instance, Stdev(0.1all) implies a situation in which manpower is equally allocated to the cost estimates of all contracts. Conversely, Stdev(0.12&0.08) implies a situation in which a contractor prioritizes the cost estimates of certain contracts.

We compared two models:

- Model(scenarios): our stochastic dynamic programming model (9), (10) and (11), and
- Model(given): model (9), (10) and (11) with assumption (4).

The scenarios of the estimated cost, E_{is}, were randomly generated by drawing samples from a normal distribution $\mathbf{N}(\mu(\tilde{E}_i), \sigma(\tilde{E}_i)^2)$ in Model(scenarios). We refer to the scenario set, $\mathcal{S}$, of the estimated cost as the training scenario set; it was used to solve the stochastic dynamic programming problem. The number of scenarios, S, in the training scenario set was 1,000, and the occurrence probability of all scenarios $s \in \mathcal{S}$ was equal, i.e., 1/1,000, in Model(scenarios). By contrast, Model(given) assumed a constant estimated cost and $E_i = \mu(\tilde{C}_i) = 1.0$ for all $i \in \mathcal{I}$.

The upper limit, U_i, on the markup was 0.5 for all $i \in \mathcal{I}$. The lower limit, L_i, was set to zero for all $i \in \mathcal{I}$ to facilitate comparison of Model(scenarios) with Model(given) in Section 4.3. On the other hand, L_i was set as (8) to investigate the effect of the VaR constraint in Section 4.4.

As mentioned in Section 3, we used (14) as the probability of winning the contract i. λ_i was set to 5 for all contacts $i \in \mathcal{I}$; that is, five competitors on average bid for each contract. κ_i and

θ_i were set to 100 and 0.012, respectively, which means that each competitor's average bid price was 1.2 and its standard deviation was 0.12.

4.2 Performance evaluation

We conducted numerical simulations to evaluate the performance of a bidding strategy. First of all, we solved a stochastic dynamic programming problem by using the training scenario set, $\mathcal{S}$, to develop an optimal strategy for determining the markup. Next, we generated 10,000 samples of

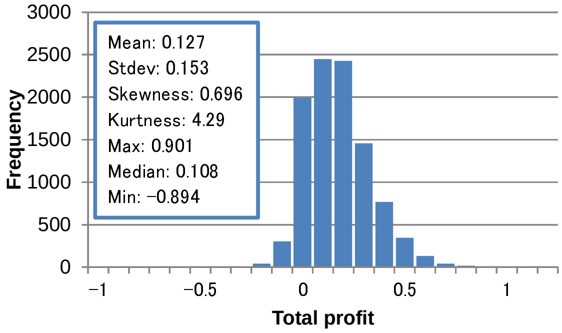
- the number of competitors who bid for the contracts $i \in \mathcal{I}$ (see also (13)),
- the bid prices which the competitors should offer for the contracts $i \in \mathcal{I}$ (see also (12)), and
- the contractor's estimated project costs of $i \in \mathcal{I}$ from $\mathbf{N}(\mu(\tilde{E}_i), \sigma(\tilde{E}_i)^2)$.

We refer to the set of these 10,000 samples as the testing scenario set; it was only used to evaluate performance of the bidding strategy. Finally, we simulated a series of competitive bids 10,000 times by using the testing scenario set. Note that the bid prices were affected by the uncertainty of the estimated costs when evaluating the performance of the bidding strategies developed by Model(scenarios) and Model(given). The simulation results for the testing scenario set are described in the following sections.

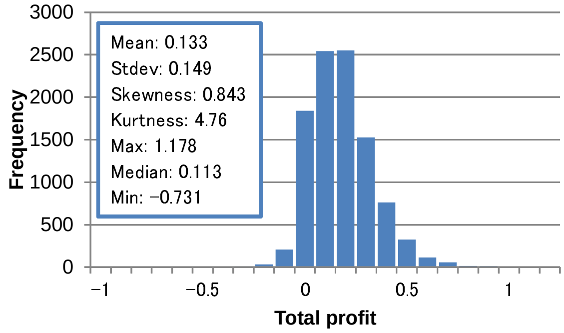
4.3 Comparison with the model based on Friedman

Let us compare our model (Model(scenarios)) with the model (Model(given)) with the lower limit, L_i, on the markup set to zero for all $i \in \mathcal{I}$. The CPU time needed for solving the stochastic dynamic programming problem (9), (10), and (11) was about 105 seconds in the case of Model(scenarios) and about 13 seconds in the case of Model(given).

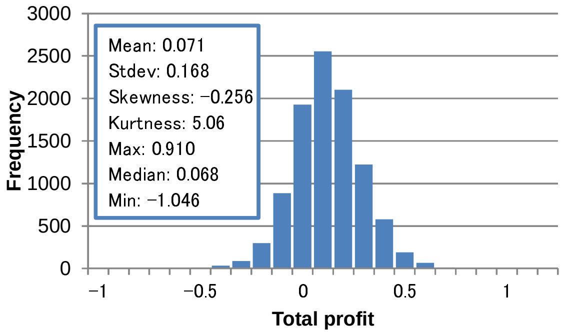
Figure 3 shows the distributions of the total profit and summary statistics. The figure indicates that the mean of the total profits obtained by Model(scenarios) was about twice that obtained by Model(given). Moreover, Model(scenarios) decreased the standard deviation (Stdev) of the total profits compared with Model(given). Since the bid prices are in practice determined from the estimated costs, the cost estimation error has a negative influence on the actual profit. Model(scenarios) takes into account the effect of the cost estimation error on the bid prices,



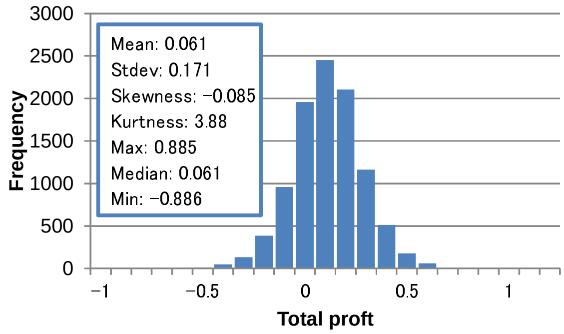
(a) Model(scenarios)-Stdev(0.1all)



(b) Model(scenarios)-Stdev(0.12&0.08)



(c) Model(given)-Stdev(0.1all)



(d) Model(given)-Stdev(0.12&0.08)

Figure 3: Distribution of the total profit

whereas this effect is completely disregarded by Model(given). Consequently, there is a marked difference in total profit between Model(scenarios) and Model(given). These observations confirm that our model is effective at both increasing the average profit and reducing the profit volatility risk.

Additionally, it can be seen that Model(scenarios) obtained a higher profit when there was a difference in the cost estimation accuracy (i.e., Stdev(0.12&0.08)) than when the cost estimation accuracies of all projects were the same (i.e., Stdev(0.1all)). This result contrasts with Model(given). As mentioned above, the estimation accuracy of the project costs can be adjusted to some extent, and accordingly, we can see that there is potential for an increase in profit if much manpower is allocated to the cost estimation of certain project contracts.

Table 1 shows the optimal markup, m_i^*, of the project contracts $i = 1, 2, 3, 4$. "Win" denotes that the corresponding contract is won, and "Lose" denotes that the corresponding contract is

Table 1: Optimal markup of the project contracts $i = 1, 2, 3, 4$

Model(scenarios)-(Stdev(0.1all)) — Project contracts

1		2		3		4
0.192	Win	---	Win	---	Win	---
					Lose	---
			Lose	0.245	Win	---
					Lose	0.178
	Lose	0.180	Win	0.245	Win	---
					Lose	0.178
			Lose	0.223	Win	---
					Lose	0.175

Model(scenarios)-(Stdev(0.12&0.08)) — Project contracts

1		2		3		4
0.262	Win	---	Win	---	Win	---
					Lose	---
			Lose	0.371	Win	---
					Lose	0.142
	Lose	0.141	Win	0.371	Win	---
					Lose	0.142
			Lose	0.334	Win	---
					Lose	0.140

Model(given)-(Stdev(0.1all)) — Project contracts

1		2		3		4
0.115	Win	---	Win	---	Win	---
					Lose	---
			Lose	0.116	Win	---
					Lose	0.091
	Lose	0.093	Win	0.116	Win	---
					Lose	0.091
			Lose	0.110	Win	---
					Lose	0.087

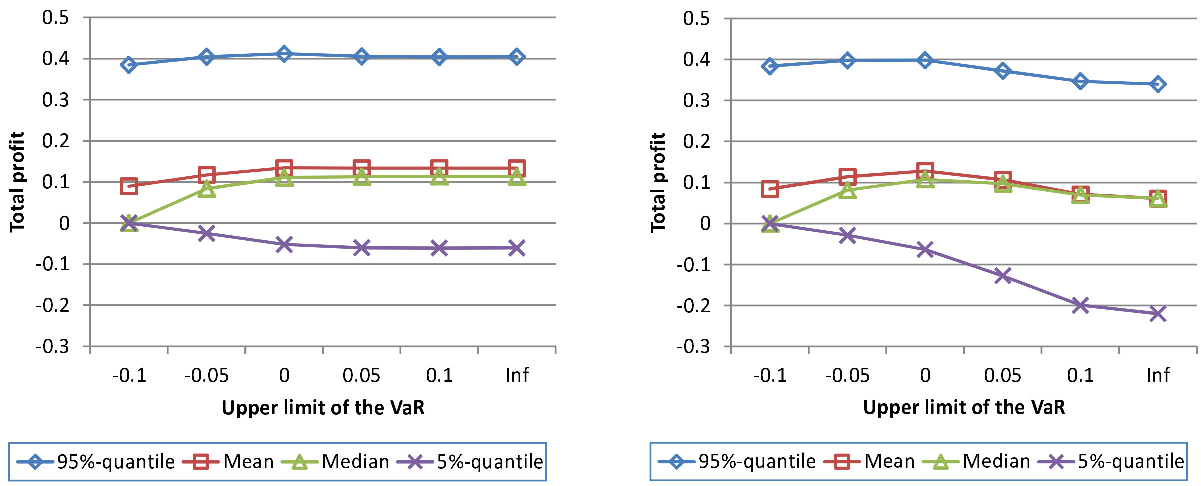
Model(given)-(Stdev(0.12&0.08)) — Project contracts

1		2		3		4
0.115	Win	---	Win	---	Win	---
					Lose	---
			Lose	0.116	Win	---
					Lose	0.091
	Lose	0.093	Win	0.116	Win	---
					Lose	0.091
			Lose	0.110	Win	---
					Lose	0.087

lost. In addition, "—" denotes that the optimal markup, m_i^*, was equal to the upper limit, U_i, which virtually means that the optimal decision is to cancel bidding for contract i. For instance, in Model(scenarios)-Stdev(0.1all), if the first and second contracts were both lost, the optimal markup of the third contract was 0.223.

Recall that the uncertainty in the estimated cost is not considered in Model(given). Accordingly, the optimal markup for Stdev(0.1all) was the same as that for Stdev(0.12&0.08) in the case of Model(given). By contrast, Model(scenarios) set the optimal markup according to the standard deviation of the estimated cost of each project. Specifically, the optimal markup of project contracts 1 and 3 for Stdev(0.12&0.08) was higher than that for Stdev(0.1all); conversely, the optimal markup of project contracts 2 and 4 for Stdev(0.1all) was higher than that for Stdev(0.12&0.08). From these results, we can see that our model sets a higher markup when the corresponding project cost is more uncertain.

Furthermore, the optimal markup of Model(scenarios) was higher than that of Model(given). This is possibly because Model(scenarios) dislikes the risk of a large loss resulting from the uncertainty about the estimated cost. A contractor might set a lower markup to ensure a



(a) Model(scenarios)-Stdev(0.12&0.08) (b) Model(given)-Stdev(0.12&0.08)

Figure 4: Effect of the VaR constraint on the total profit

successful bid if s/he wants more profit. However, these observations confirm that a higher markup can lead to even higher long-term profit when there is uncertainty about the estimated cost.

A rolling-horizon implementation is an effective way of putting our bidding strategy to practical use. Specifically, we first make a long-term bidding plan and solve our stochastic dynamic programming model. However, plans are subject to change. For instance, a scheduled bidding session might be canceled, or the contractor's bidding policy might change. This is why we should bid for only the first project contract and then solve our stochastic dynamic programming model once again in view of possible changes in plan. Repeating this process, i.e., the rolling-horizon implementation, will surely enhance the effectiveness of our model in a real situation.

4.4 Effect of the VaR constraint

In this section, we investigate the effect of the VaR constraint (see also (8)) on the total profit. Here, the confidence level, β, was set to 0.95, and the upper limit of the VaR, α_i, was chosen from $\{-0.1, -0.05, 0, 0.05, 0.1\}$ for all $i \in \mathcal{I}$.

Figure 4 shows four kinds of total profit statistics, i.e., 95%-quantile, mean, median and 5%-quantile of the total profit. Note that "Inf" means that $\alpha_i = \infty$ for all $i \in \mathcal{I}$, which leads to the same results as in Section 4.3.

Figure 4 shows that the VaR constraint had a relatively small effect on the result of Model(scenarios) when α_i ranged from 0 to ∞. This is because Model(scenarios) accounts for the uncertainty about the estimated cost through the expected profit (3). By contrast, the VaR constraint was quite useful for Model(given). We can see from Figure 4 that the 5%-quantile of the total profits significantly improved by decreasing α_i from ∞ to 0. Specifically, the 5%-quantile of the total profits was -0.220 in the case of $\alpha_i = \infty$, and it was -0.063 in the case of $\alpha_i = 0$. In addition, it is noteworthy that the other statistics of Model(given) also improved as a result of decreasing α_i from ∞ to 0. In particular, the statistics of Model(given) with $\alpha_i = 0$ were similar to those of Model(scenarios) with $\alpha_i = \infty$.

These observations suggest that by imposing the VaR constraint, Model(given), which disregards the uncertainty about the estimated cost, can realize a profit that is comparable with the profit of our model (Model(scenarios)). However, we still think that Model(scenarios) is more useful than Model(given) with a VaR constraint. This is because the confidence level, β, and the upper limit of the VaR, α_i, need to be set appropriately. Additionally, the VaR constraint (see also (8)) is based on the assumption that the estimated cost, $\tilde{E}_i$, follows a normal distribution. Let us suppose that the estimated cost is not normally distributed. It might be that the VaR constraint, i.e., the lower limit (8), still has a beneficial effect; however, there is no theoretical guarantee. On the other hand, we only have to generate scenarios, E_{is}, of the estimated cost from the probability distribution we want in order to ensure that Model(scenarios) will be effective.

5 Conclusions

We developed a new stochastic dynamic programming model for establishing a strategy for sequential competitive bidding. The well-known Friedman's model disregards the uncertainty in the estimated cost. However, the bid price is usually set by putting a markup on the estimated cost, and consequently, the bid price is affected by inaccurate cost estimates. We took into account the effect of inaccurate cost estimates on the bid price by utilizing a scenario-based approach. Moreover, we employed the value-at-risk (VaR) constraint to mitigate the risk of a large loss.

We conducted numerical experiments to assess the effectiveness of our model. The results showed that higher profit can be obtained by setting the markup according to the estimation accuracy of each project cost. In addition, we found that the VaR constraint can improve on the profit obtained by the Friedman's model.

One direction of further study is to develop a risk-averse sequential bidding strategy by employing what is called a dynamic risk measure (see Ruszczyński, 2010, and the references therein). Although we imposed the VaR constraint on each project contract, the use of the dynamic risk measure, which was developed for dynamic programming problems, might be more effective. Another direction of study is to optimize the manpower allocation to the cost estimation of each project, as is done in Ishii et al. (2011). However, both directions of study lead us to a dynamic programming recursion which is difficult to handle.

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